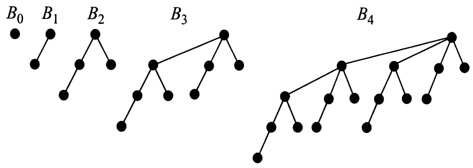
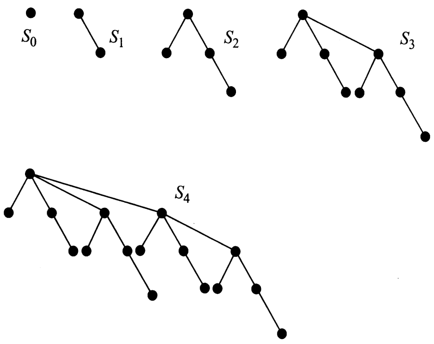
13.



15. Because B_{k+1} is formed from two copies of B_k, one shifted down one level, the height increases by 1 as k increases by 1. Because B_0 had height 0, it follows by induction that B_k has height k. **17.** Because the root of B_{k+1} is the root of B_k with one additional child (namely the root of the other B_k), the degree of the root increases by 1 as k increases by 1. Because B_0 had a root with degree 0, it follows by induction that B_k has a root with degree k.

19.

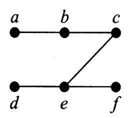


21. Use mathematical induction. The result is trivial for $k = 0$. Suppose it is true for $k - 1$. T_{k-1} is the parent tree for T. By induction, the child tree for T can be obtained from $T_0, \ldots, T_{k-2}$ in the manner stated. The final connection of r_{k-2} to r_{k-1} is as stated in the definition of S_k-tree.

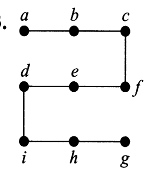
23. **procedure** *level*(T: ordered rooted tree with root r)
queue := sequence consisting of just the root r
while *queue* contains at least one term
begin
  v := first vertex in queue
  list v
  remove v from queue and put children of v onto the end of queue
end

25. Build the tree by inserting a root for the address 0, and then inserting a subtree for each vertex labeled i, for i a positive integer, built up from subtrees for each vertex labeled $i.j$ for j a positive integer, and so on. **27. a)** Yes **b)** No **c)** Yes
29. The resulting graph has no edge that is in more than one simple circuit of the type described. Hence, it is a cactus.

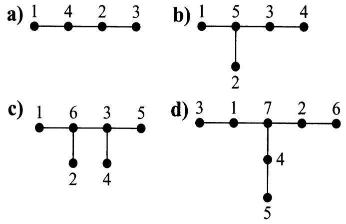
31.



33.



35.



37. 6 **39. a)** 0 for 00, 11 for 01, 100 for 10, 101 for 11 (exact coding depends on how ties were broken, but all versions are equivalent); $0.645n$ for string of length n **b)** 0 for 000, 100 for 001, 101 for 010, 110 for 100, 11100 for 011, 11101 for 101, 11110 for 110, 11111 for 111 (exact coding depends on how ties were broken, but all versions are equivalent); $0.532\overline{6}n$ for string of length n **41.** Let G' be the graph obtained by deleting from G the vertex v and all edges incident to v. A minimum spanning tree of G can be obtained by taking an edge of minimal weight incident to v together with a minimum spanning tree of G'. **43.** Suppose that edge e is the edge of least weight incident to vertex v, and suppose that T is a spanning tree that does not include e. Add e to T, and delete from the simple circuit formed thereby the other edge of the circuit that contains v. The result will be a spanning tree of strictly smaller weight (because the deleted edge has weight greater than the weight of e). This is a contradiction, so T must include e.

CHAPTER 11

Section 11.1

1. a) 1 **b)** 1 **c)** 0 **d)** 0 **3. a)** $(1 \cdot 1) + (\overline{0 \cdot 1} + 0) = 1 + (\overline{0} + 0) = 1 + (1 + 0) = 1 + 1 = 1$
b) $(\mathbf{T} \wedge \mathbf{T}) \vee (\neg(\mathbf{F} \wedge \mathbf{T}) \vee \mathbf{F}) \equiv \mathbf{T}$

5. a)

x	y	z	$x\bar{y}$
1	1	1	0
1	1	0	0
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	1
0	0	1	0
0	0	0	0

b)

x	y	z	$x + yz$
1	1	1	1
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	0

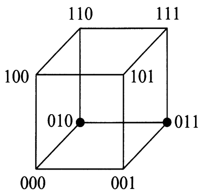
c)

x	y	z	$x\bar{y} + \overline{xyz}$
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	1
0	1	1	1
0	1	0	1
0	0	1	1
0	0	0	1

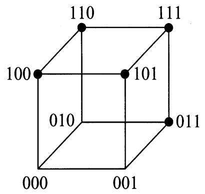
d)

x	y	z	$x(yz + \bar{y}\,\bar{z})$
1	1	1	1
1	1	0	0
1	0	1	0
1	0	0	1
0	1	1	0
0	1	0	0
0	0	1	0
0	0	0	0

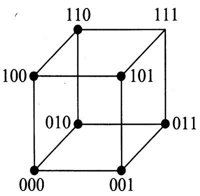
7. a)



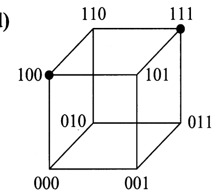
b)



c)



d)



9. (0, 0) and (1, 1) **11.** $x + xy = x \cdot 1 + xy = x(1 + y) = x(y + 1) = x \cdot 1 = x$

13.

x	y	z	$x\bar{y}$	$y\bar{z}$	$\bar{x}z$	$x\bar{y} + y\bar{z} + \bar{x}z$	$\bar{x}y$	$\bar{y}z$	$x\bar{z}$	$\bar{x}y + \bar{y}z + x\bar{z}$
1	1	1	0	0	0	0	0	0	0	0
1	1	0	0	1	0	1	0	0	1	1
1	0	1	1	0	0	1	0	1	0	1
1	0	0	1	0	0	1	0	0	1	1
0	1	1	0	0	1	1	1	0	0	1
0	1	0	0	1	0	1	1	0	0	1
0	0	1	0	0	1	1	0	1	0	1
0	0	0	0	0	0	0	0	0	0	0

15.

x	$x + x$	$x \cdot x$
0	0	0
1	1	1

17.

x	$x + 1$	$x \cdot 0$
0	1	0
1	1	0

19.

x	y	z	$y + z$	$x + (y + z)$	$x + y$	$(x + y) + z$	yz	$x(yz)$	xy	$(xy)z$
1	1	1	1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	0	0	1	0
1	0	1	1	1	1	1	0	0	0	0
1	0	0	0	1	1	1	0	0	0	0
0	1	1	1	1	1	1	1	0	0	0
0	1	0	1	1	1	1	0	0	0	0
0	0	1	1	1	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

21.

x	y	xy	$\overline{(xy)}$	$\bar{x}$	$\bar{y}$	$\bar{x} + \bar{y}$	$x + y$	$\overline{(x + y)}$	$\bar{x}\bar{y}$
1	1	1	0	0	0	0	1	0	0
1	0	0	1	0	1	1	1	0	0
0	1	0	1	1	0	1	1	0	0
0	0	0	1	1	1	1	0	1	1

23. $0 \cdot \bar{0} = 0 \cdot 1 = 0$; $1 \cdot \bar{1} = 1 \cdot 0 = 0$

25.

x	y	$x \oplus y$	$x + y$	xy	$\overline{(xy)}$	$(x + y)\overline{(xy)}$	$x\bar{y}$	$\bar{x}y$	$x\bar{y} + \bar{x}y$
1	1	0	1	1	0	0	0	0	0
1	0	1	1	0	1	1	1	0	1
0	1	1	1	0	1	1	0	1	1
0	0	0	0	0	1	0	0	0	0

27. a) True, as a table of values can show **b)** False; take $x = 1, y = 1, z = 1$, for instance **c)** False; take $x = 1, y = 1, z = 0$, for instance **29.** By De Morgan's laws, the complement of an expression is like the dual except that the complement of each variable has been taken. **31.** 16 **33.** If we replace each 0 by **F**, 1 by **T**, Boolean sum by $\vee$, Boolean product by $\wedge$, and $\bar{}$ by $\neg$ (and x by p and y by q so that the variables look like they represent propositions, and the equals sign by the logical equivalence symbol), then $\overline{xy} = \bar{x} + \bar{y}$ becomes $\neg(p \wedge q) \equiv \neg p \vee \neg q$ and $\overline{x + y} = \bar{x}\,\bar{y}$ becomes $\neg(p \vee q) \equiv \neg p \wedge \neg q$. **35.** By the domination, distributive, and identity laws, $x \vee x = (x \vee x) \wedge 1 = (x \vee x) \wedge (x \vee \bar{x}) = x \vee (x \wedge \bar{x}) = x \vee 0 = x$. Similarly, $x \wedge x = (x \wedge x) \vee 0 = (x \wedge x) \vee (x \wedge \bar{x}) = x \wedge (x \vee \bar{x}) = x \wedge 1 = x$. **37.** Because $0 \vee 1 = 1$ and $0 \wedge 1 = 0$ by the identity and commutative laws, it follows that $\bar{0} = 1$. Similarly, because $1 \vee 0 = 1$ and $1 \wedge 0 = 1$, it follows that $\bar{1} = 0$. **39.** First, note that $x \wedge 0 = 0$ and $x \vee 1 = 1$ for all x, as can easily be proved. To prove the first identity, it is sufficient to show that $(x \vee y) \vee (\bar{x} \wedge \bar{y}) = 1$ and $(x \vee y) \wedge (\bar{x} \wedge \bar{y}) = 0$. By the associative, commutative, distributive, domination, and identity laws, $(x \vee y) \vee (\bar{x} \wedge \bar{y}) = y \vee [x \vee (\bar{x} \wedge \bar{y})] = y \vee [(x \vee \bar{x}) \wedge (x \vee \bar{y})] = y \vee [1 \wedge (x \vee \bar{y})] = y \vee (x \vee \bar{y}) = (y \vee \bar{y}) \vee x = 1 \vee x = 1$ and $(x \vee y) \wedge (\bar{x} \wedge \bar{y}) = \bar{y} \wedge [\bar{x} \wedge (x \vee y)] = \bar{y} \wedge [(\bar{x} \wedge x) \vee (\bar{x} \wedge y)] = \bar{y} \wedge [0 \vee (\bar{x} \wedge y)] = \bar{y} \wedge (\bar{x} \wedge y) = \bar{x} \wedge (y \wedge \bar{y}) = \bar{x} \wedge 0 = 0$. The second identity is proved in a similar way. **41.** Using the hypotheses, Exercise 35, and the distributive law it follows that $x = x \vee 0 = x \vee (x \vee y) = (x \vee x) \vee y = x \vee y = 0$. Similarly, $y = 0$. To prove the second statement, note that $x = x \wedge 1 = x \wedge (x \wedge y) = (x \wedge x) \wedge y = x \wedge y = 1$. Similarly, $y = 1$. **43.** Use Exercises 39 and 41 in the Supplementary Exercises in Chapter 8 and the definition of a complemented, distributed lattice to establish the five pairs of laws in the definition.

Section 11.2

1. a) $\bar{x}\,\bar{y}z$ **b)** $\bar{x}y\bar{z}$ **c)** $\bar{x}yz$ **d)** $\overline{xyz}$ **3. a)** $xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\,\bar{z} + \bar{x}yz + \bar{x}y\bar{z} + \bar{x}\,\bar{y}z$ **b)** $xyz + xy\bar{z} + \bar{x}yz$ **c)** $xyz + xy\bar{z} + x\bar{y}z + x\bar{y}\,\bar{z}$ **d)** $x\bar{y}z + x\bar{y}\,\bar{z}$ **5.** $wxy\bar{z} + wx\bar{y}z + w\bar{x}yz + \bar{w}xyz + \bar{w}x\bar{y}\,\bar{z} + \bar{w}\,\bar{x}\,\bar{y}z + \bar{w}\,\bar{x}y\bar{z} + w\bar{x}\,\bar{y}\,\bar{z}$ **7. a)** $\bar{x} + \bar{y} + z$ **b)** $x + y + z$ **c)** $x + \bar{y} + z$ **9.** $y_1 + y_2 + \cdots + y_n = 0$ if and only if $y_i = 0$ for $i = 1, 2, \ldots, n$. This holds if and only if $x_i = 0$ when $y_i = x_i$ and $x_i = 1$ when $y_i = \bar{x}_i$. **11. a)** $x + y + z$ **b)** $(x + y + z)(x + y + \bar{z})(x + \bar{y} + z)(\bar{x} + y + z)(\bar{x} + y + \bar{z})$ **c)** $(x + y + z)(x + y + \bar{z})(x + \bar{y} + z)(x + \bar{y} + \bar{z})$ **d)** $(x + y + z)(x + y + \bar{z})(x + \bar{y} + z)(x + \bar{y} + \bar{z})(\bar{x} + \bar{y} + z)(\bar{x} + \bar{y} + \bar{z})$

13. a) $x+y+z$ **b)** $x+\overline{[y+\overline{(\overline{x}+z)}]}$ **c)** $\overline{(x+\overline{y})}$
d) $[x+\overline{(x+\overline{y}+\overline{z})}]$

15. a)

x	$\overline{x}$	$x \downarrow x$
1	0	0
0	1	1

b)

x	y	xy	$x \downarrow x$	$y \downarrow y$	$(x \downarrow x) \downarrow (y \downarrow y)$
1	1	1	0	0	1
1	0	0	0	1	0
0	1	0	1	0	0
0	0	0	1	1	0

c)

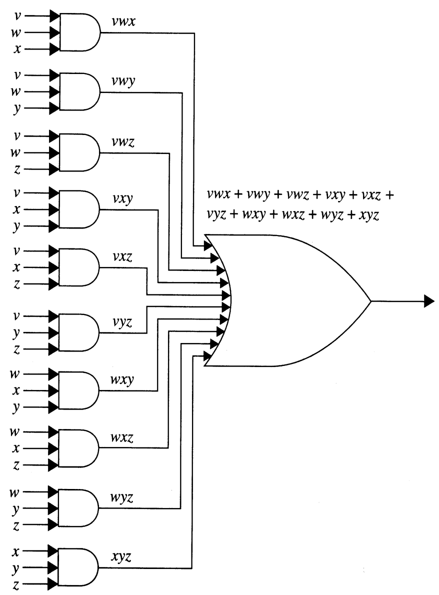
x	y	$x+y$	$(x \downarrow y)$	$(x \downarrow y) \downarrow (x \downarrow y)$
1	1	1	0	1
1	0	1	0	1
0	1	1	0	1
0	0	0	1	0

17. a) $\{[(x \mid x) \mid (y \mid y)] \mid [(x \mid x) \mid (y \mid y)]\} \mid (z \mid z)$ **b)** $\{[(x \mid x) \mid (z \mid z)] \mid y\} \mid \{[(x \mid x) \mid (z \mid z)] \mid y\}$ **c)** x **d)** $[x \mid (y \mid y)] \mid [x \mid (y \mid y)]$ **19.** It is impossible to represent $\overline{x}$ using $+$ and $\cdot$ because there is no way to get the value 0 if the input is 1.

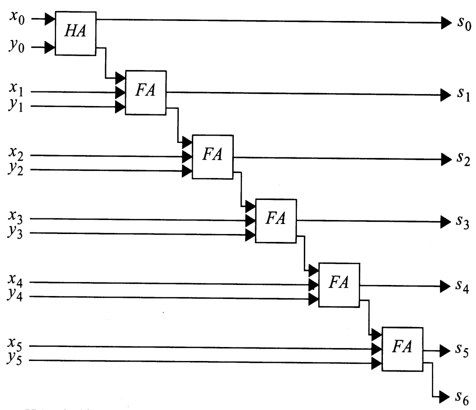
Section 11.3

1. $(x+y)\overline{y}$ **3.** $\overline{(xy)}+(\overline{z}+x)$ **5.** $(x+y+z)+(\overline{x}+y+z)+(\overline{x}+\overline{y}+\overline{z})$

7.

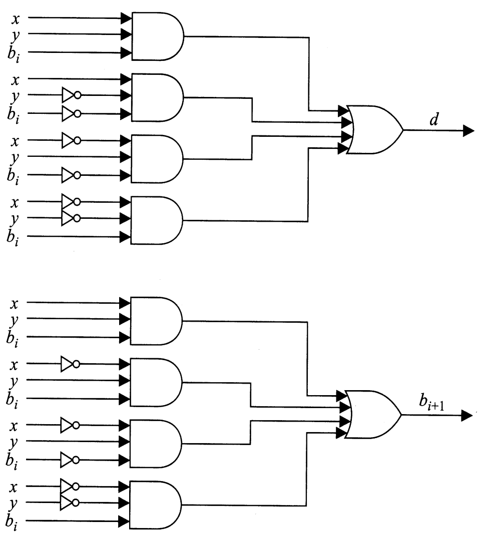


9.

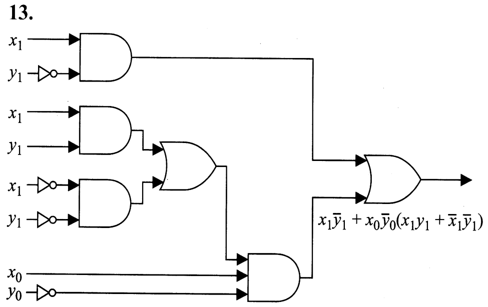


HA = half adder
FA = full adder

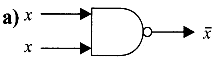
11.



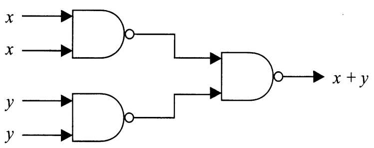
13.



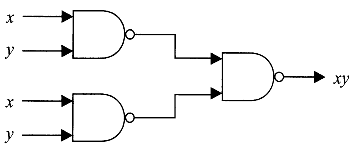
15. a)



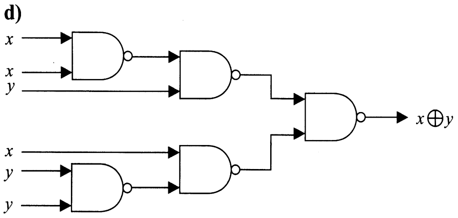
b)



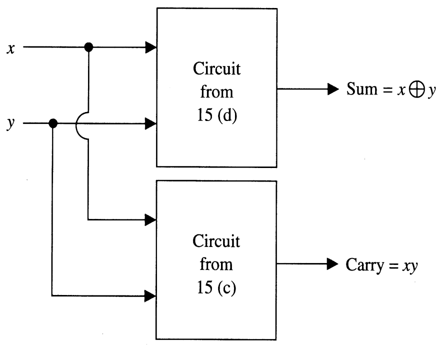
c)



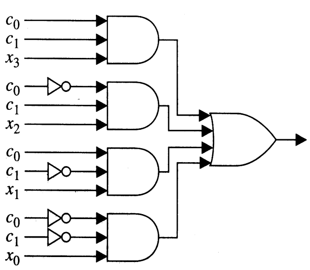
d)



17.



19.



Section 11.4

1. a)

	y	$\bar{y}$
x		
$\bar{x}$	1	

b) xy and $\overline{x}\,\overline{y}$

3. a)

	y	$\bar{y}$
x		1
$\bar{x}$		

b)

	y	$\bar{y}$
x	1	
$\bar{x}$		1

c)

	y	$\bar{y}$
x	1	1
$\bar{x}$	1	1

5. a)

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
x				
$\bar{x}$		1		

b) $\bar{x}yz, \bar{x}\,\bar{y}\,\bar{z}, xy\bar{z}$

7. a)

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
x			1	
$\bar{x}$				

b)

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
x				
$\bar{x}$	1		1	

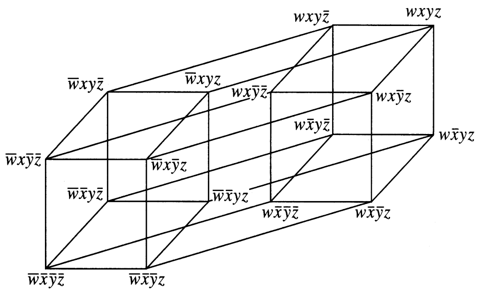
c)

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
x	1	1		1
$\bar{x}$				1

9. Implicants: xyz, $xy\bar{z}$, $x\,\bar{y}\,\bar{z}$, $\bar{x}y\bar{z}$, xy, $x\bar{z}$, $y\bar{z}$; prime implicants: xy, $x\bar{z}$, $y\bar{z}$; essential prime implicants: xy, $x\bar{z}$, $y\bar{z}$

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
x	1	1	1	
$\bar{x}$		1		

11. The 3-cube on the right corresponds to w; the 3-cube given by the top surface of the whole figure represents x; the 3-cube given by the back surface of the whole figure represents y; the 3-cube given by the right surfaces of both the left and the right 3-cube represents z. In each case, the opposite 3-face represents the complemented literal. The 2-cube that represents wz is the right face of the 3-cube on the right; the 2-cube that represents $\bar{x}y$ is bottom rear; the 2-cube that represents $\bar{y}\,\bar{z}$ is front left.



13. a)

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
wx				
$w\bar{x}$				
$\bar{w}\bar{x}$				
$\bar{w}x$		1		

b) $\bar{w}xyz$, $\bar{w}\,\bar{x}yz$, $\bar{w}x\bar{y}\,\bar{z}$, $\bar{w}x\bar{y}\,\bar{z}$

15. a)

	$x_3x_4x_5$	$x_3x_4\bar{x}_5$	$x_3\bar{x}_4\bar{x}_5$	$x_3\bar{x}_4x_5$	$\bar{x}_3\bar{x}_4x_5$	$\bar{x}_3\bar{x}_4\bar{x}_5$	$\bar{x}_3x_4\bar{x}_5$	$\bar{x}_3x_4x_5$
x_1x_2	1	1						
$x_1\bar{x}_2$								
$\bar{x}_1\bar{x}_2$								
$\bar{x}_1x_2$								

b)

	$x_3x_4x_5$	$x_3x_4\bar{x}_5$	$x_3\bar{x}_4\bar{x}_5$	$x_3\bar{x}_4x_5$	$\bar{x}_3\bar{x}_4x_5$	$\bar{x}_3\bar{x}_4\bar{x}_5$	$\bar{x}_3x_4\bar{x}_5$	$\bar{x}_3x_4x_5$
x_1x_2								
$x_1\bar{x}_2$								
$\bar{x}_1\bar{x}_2$	1			1				
$\bar{x}_1x_2$	1			1				

c)

	$x_3x_4x_5$	$x_3x_4\bar{x}_5$	$x_3\bar{x}_4\bar{x}_5$	$x_3\bar{x}_4x_5$	$\bar{x}_3\bar{x}_4x_5$	$\bar{x}_3\bar{x}_4\bar{x}_5$	$\bar{x}_3x_4\bar{x}_5$	$\bar{x}_3x_4x_5$
x_1x_2	1	1					1	1
$x_1\bar{x}_2$								
$\bar{x}_1\bar{x}_2$								
$\bar{x}_1x_2$	1	1					1	1

d)

	$x_3x_4x_5$	$x_3x_4\bar{x}_5$	$x_3\bar{x}_4\bar{x}_5$	$x_3\bar{x}_4x_5$	$\bar{x}_3\bar{x}_4x_5$	$\bar{x}_3\bar{x}_4\bar{x}_5$	$\bar{x}_3x_4\bar{x}_5$	$\bar{x}_3x_4x_5$
x_1x_2					1	1		
$x_1\bar{x}_2$					1	1		
$\bar{x}_1\bar{x}_2$					1	1		
$\bar{x}_1x_2$					1	1		

e)

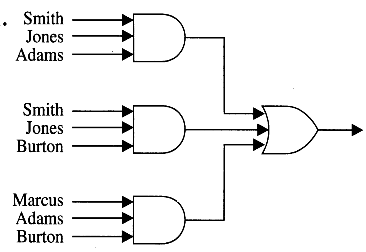
	$x_3x_4x_5$	$x_3x_4\bar{x}_5$	$x_3\bar{x}_4\bar{x}_5$	$x_3\bar{x}_4x_5$	$\bar{x}_3\bar{x}_4x_5$	$\bar{x}_3\bar{x}_4\bar{x}_5$	$\bar{x}_3x_4\bar{x}_5$	$\bar{x}_3x_4x_5$
x_1x_2	1	1	1	1				
$x_1\bar{x}_2$	1	1	1	1				
$\bar{x}_1\bar{x}_2$	1	1	1	1				
$\bar{x}_1x_2$	1	1	1	1				

f)

	$x_3x_4x_5$	$x_3x_4\bar{x}_5$	$x_3\bar{x}_4\bar{x}_5$	$x_3\bar{x}_4x_5$	$\bar{x}_3\bar{x}_4x_5$	$\bar{x}_3\bar{x}_4\bar{x}_5$	$\bar{x}_3x_4\bar{x}_5$	$\bar{x}_3x_4x_5$
x_1x_2		1	1			1	1	
$x_1\bar{x}_2$		1	1			1	1	
$\bar{x}_1\bar{x}_2$		1	1			1	1	
$\bar{x}_1x_2$		1	1			1	1	

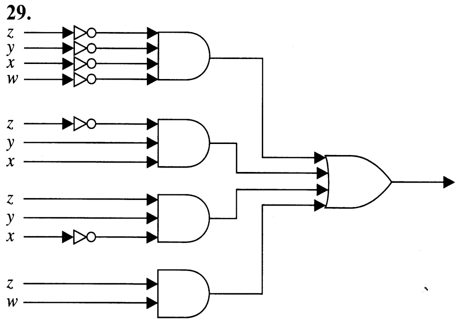
17. a) 64 **b)** 6 **19.** Rows 1 and 4 are considered adjacent. The pairs of columns considered adjacent are: columns 1 and 4, 1 and 12, 1 and 16, 2 and 11, 2 and 15, 3 and 6, 3 and 10, 4 and 9, 5 and 8, 5 and 16, 6 and 15, 7 and 10, 7 and 14, 8 and 13, 9 and 12, 11 and 14, 13 and 16.

21.



23. a) $\bar{x}z$ **b)** y **c)** $x\bar{z}+\bar{x}z+\bar{y}z$ **d)** $xz+\bar{x}y+\bar{y}\bar{z}$ **25. a)** $wxz+wx\bar{y}+w\bar{y}z+w\bar{x}y\bar{z}$ **b)** $x\bar{y}z+\bar{w}\bar{y}z+wxy\bar{z}+w\bar{x}yz+\bar{w}\bar{x}y\bar{z}$ **c)** $\bar{y}z+wxy+w\bar{x}\bar{y}+\bar{w}\bar{x}y\bar{z}$ **d)** $wy+yz+\bar{x}y+wxz+\bar{w}\bar{x}z$ **27.** $x(y+z)$

29.



31. $\bar{x}\bar{z}+xz$ **33.** We use induction on n. If $n=1$, then we are looking at a line segment, labeled 0 at one end and 1 at the other end. The only possible value of k is also 1, and if the literal is x_1, then the subcube we have is the 0-dimensional subcube consisting of the endpoint labeled 1, and if the literal is $\bar{x}_1$, then the subcube we have is the 0-dimensional subcube consisting of the endpoint labeled 0. Now assume that the statement is true for n; we must show that it is true for $n+1$. If the literal x_{n+1} (or its complement) is not part of the product, then by the inductive hypothesis, the product when viewed in the setting of n variables corresponds to an $(n-k)$-dimensional subcube of the n-dimensional cube, and the Cartesian product of that subcube with the line segment [0, 1] gives us a subcube one dimension higher in our given $(n+1)$-dimensional cube, namely having dimension $(n+1)-k$, as desired. On the other hand, if the literal x_{n+1} (or its complement) is part of the product, then the product of the remaining $k-1$ literals corresponds to a subcube of dimension $n-(k-1)=(n+1)-k$ in the n-dimensional cube, and that slice, at either the 1-end or the 0-end in the last variable, is the desired subcube.

Supplementary Exercises

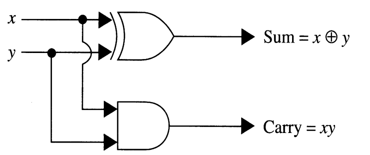
1. a) $x=0,\ y=0,\ z=0;\ x=1,\ y=1,\ z=1$ **b)** $x=0,\ y=0, z=0; x=0, y=0, z=1; x=0, y=1, z=0; x=1, y=0,\ z=1;\ x=1,\ y=1,\ z=0;\ x=1,\ y=1,\ z=1$ **c)** No values **3. a)** Yes **b)** No **c)** No **d)** Yes **5.** $2^{2^{n-1}}$ **7. a)** If $F(x_1,\ldots,x_n)=1$, then $(F+G)(x_1,\ldots,x_n)=F(x_1,\ldots,x_n)+G(x_1,\ldots,x_n)=1$ by the dominance law. Hence, $F\le F+G$. **b)** If $(FG)(x_1,\ldots,x_n)=1$, then $F(x_1,\ldots,x_n)\cdot G(x_1,\ldots,x_n)=1$. Hence, $F(x_1,\ldots,x_n)=1$. It follows that $FG\le F$. **9.** Because $F(x_1,\ldots,x_n)=1$ implies that $F(x_1,\ldots,x_n)=1$, $\le$ is reflexive. Suppose that $F\le G$ and $G\le F$. Then $F(x_1,\ldots,x_n)=1$ if and only if $G(x_1,\ldots,x_n)=1$. This implies that $F=G$. Hence, $\le$ is antisymmetric. Suppose that $F\le G\le H$. Then if $F(x_1,\ldots,x_n)=1$, it follows that $G(x_1,\ldots,x_n)=1$, which implies that $H(x_1,\ldots,x_n)=1$. Hence, $F\le H$, so that $\le$ is transitive. **11. a)** $x=1, y=0, z=0$ **b)** $x=1,\ y=0, z=0$ **c)** $x=1, y=0, z=0$

13.

x	y	$x \odot y$	$x \oplus y$	$\overline{(x \oplus y)}$
1	1	1	0	1
1	0	0	1	0
0	1	0	1	0
0	0	1	0	1

15. Yes, as a truth table shows **17. a)** 6 **b)** 5 **c)** 5 **d)** 6

19.



21. $x_3 + x_2\overline{x_1}$ **23.** Suppose it were with weights a and b. Then there would be a real number T such that $xa + yb \geq T$ for (1,0) and (0,1), but with $xa + yb < T$ for (0,0) and (1,1). Hence, $a \geq T, b \geq T, 0 < T$, and $a + b < T$. Thus, a and b are positive, which implies that $a + b > a \geq T$, a contradiction.

CHAPTER 12

Section 12.1

1. a) sentence $\Rightarrow$ **noun phrase intransitive verb phrase** $\Rightarrow$ **article adjective noun intransitive verb phrase** $\Rightarrow$ **article adjective noun intransitive verb** $\Rightarrow \ldots$ (after 3 steps) $\ldots \Rightarrow$ *the happy hare runs.* **b) sentence** $\Rightarrow$ **noun phrase intransitive verb phrase** $\Rightarrow$ **article adjective noun intransitive verb phrase** $\Rightarrow$ **article adjective noun intransitive verb adverb**... (after 4 steps)... $\Rightarrow$ *the sleepy tortoise runs quickly* **c) sentence** $\Rightarrow$ **noun phrase transitive verb phrase noun phrase** $\Rightarrow$ **article noun transitive verb phrase noun phrase** $\Rightarrow$ **article noun transitive verb noun phrase** $\Rightarrow$ **article noun transitive verb article noun** $\Rightarrow \ldots$(after 4 steps)... $\Rightarrow$ *the tortoise passes the hare* **d) sentence** $\Rightarrow$ **noun phrase transitive verb phrase noun phrase** $\Rightarrow$ **article adjective noun transitive verb phrase noun phrase** $\Rightarrow$ **article adjective noun transitive verb noun phrase** $\Rightarrow$ **article adjective noun transitive verb article adjective noun** $\Rightarrow \ldots$(after 6 steps)... $\Rightarrow$ *the sleepy hare passes the happy tortoise*

3. The only way to get a noun, such as *tortoise*, at the end is to have a noun phrase at the end, which can be achieved only via the production **sentence** $\rightarrow$ **noun phrase transitive verb phrase noun phrase**. However, **transitive verb phrase** $\rightarrow$ **transitive verb** $\rightarrow$ *passes*, and this sentence does not contain *passes*.
5. a) $S \Rightarrow 1A \Rightarrow 10B \Rightarrow 101A \Rightarrow 1010B \Rightarrow 10101$ **b)** Because of the productions in this grammar, every 1 must be followed by a 0 unless it occurs at the end of the string. **c)** All strings consisting of a 0 or a 1 followed by one or more repetitions of 01
7. $S \Rightarrow 0S1 \Rightarrow 00S11 \Rightarrow 000S111 \Rightarrow 000111$
9. a) $S \Rightarrow 0S \Rightarrow 00S \Rightarrow 00S1 \Rightarrow 00S11 \Rightarrow 00S111 \Rightarrow 00S1111 \Rightarrow 001111$ **b)** $S \Rightarrow 0S \Rightarrow 00S \Rightarrow 001A \Rightarrow 0011A \Rightarrow 00111A \Rightarrow 001111$ **11.** $S \Rightarrow 0SAB \Rightarrow 00SABAB \Rightarrow 00ABAB \Rightarrow 00AABB \Rightarrow 001ABB \Rightarrow 0011BB \Rightarrow 00112B \Rightarrow 001122$
13. a) $S \rightarrow 0, S \rightarrow 1, S \rightarrow 11$ **b)** $S \rightarrow 1S, S \rightarrow \lambda$ **c)** $S \rightarrow 0A1,\ A \rightarrow 1A,\ A \rightarrow 0A,\ A \rightarrow \lambda$ **d)** $S \rightarrow 0A, A \rightarrow 11A, A \rightarrow \lambda$ **15. a)** $S \rightarrow 00S, S \rightarrow \lambda$ **b)** $S \rightarrow 10A, A \rightarrow 00A, A \rightarrow \lambda$ **c)** $S \rightarrow AAS, S \rightarrow BBS, AB \rightarrow BA, BA \rightarrow AB, S \rightarrow \lambda, A \rightarrow 0, B \rightarrow 1$ **d)** $S \rightarrow 0000000000A, A \rightarrow 0A, A \rightarrow \lambda$ **e)** $S \rightarrow AS, S \rightarrow ABS, S \rightarrow A, AB \rightarrow BA, BA \rightarrow AB, A \rightarrow 0, B \rightarrow 1$ **f)** $S \rightarrow ABS, S \rightarrow \lambda, AB \rightarrow BA, BA \rightarrow AB, A \rightarrow 0, B \rightarrow 1$ **g)** $S \rightarrow ABS, S \rightarrow T, S \rightarrow U, T \rightarrow AT, T \rightarrow A, U \rightarrow BU, U \rightarrow B, AB \rightarrow BA, BA \rightarrow AB, A \rightarrow 0, B \rightarrow 1$ **17. a)** $S \rightarrow 0S, S \rightarrow \lambda$ **b)** $S \rightarrow A0, A \rightarrow 1A, A \rightarrow \lambda$ **c)** $S \rightarrow 000S, S \rightarrow \lambda$ **19. a)** Type 2, not type 3 **b)** Type 3 **c)** Type 0, not type 1 **d)** Type 2, not type 3 **e)** Type 2, not type 3 **f)** Type 0, not type 1 **g)** Type 3 **h)** Type 0, not type 1 **i)** Type 2, not type 3 **j)** Type 2, not type 3 **21.** Let S_1 and S_2 be the start symbols of G_1 and G_2, respectively. Let S be a new start symbol. **a)** Add S and productions $S \rightarrow S_1$ and $S \rightarrow S_2$. **b)** Add S and production $S \rightarrow S_1S_2$. **c)** Add S and production $S \rightarrow \lambda$ and $S \rightarrow S_1S$.

23. a)